TECHNOLOGY
TANDEM

GSDMC
HIGH SCHOOL
MATH FIELD DAY

MARCH 8, 2008

Sponsored by:
CUYAMACA COMMUNITY COLLEGE

INSTRUCTIONS:

You may work in pairs and field a maximum of two teams per school.

We will supply each team member with a TI-84 Graphing Calculator. Please leave your personal calculators at the front of the room.

Work as many of the problems as you can in the time allotted. You should not expect to finish every problem, so choose the ones you can readily complete.

The number of points earned will determine the winners. Be aware there is **NO penalty for guessing**.

The judges must score some of the problems on the spot. In this case, one team member will take the calculator to the appropriate judge for immediate grading. The judge will note the score for that question and will add it to your team score after the competition is over.

In order to improve this event for next year's competition, we would appreciate your feedback. Thanks for participating!

You may keep this test to prepare for next year's Technology Tandem competition, as the contents of the exam may be similar to this year's.

Each answer is worth five points.

1. Find the prime factorization of 440510115317719599.

2. Find the prime factorization of 469462244429724416.

3. Find the prime factorization of 111017469522094069360684209050360988682614703.

4. Find the smallest pair of twin primes greater than 200.

5. Find the smallest pair of twin primes greater than 9500.

6. Find the next pair of twin primes greater than 9500 (i.e. the first pair greater than the pair found in the previous problem).

7. Given that 74997941 and 74997943 are twin primes, find the next pair of twin primes.

8. Find the greatest common factor of 79070145246 and 85968677917507.

9. Find the least common multiple of 3145149 and 22818509.

10. Does $\sin\left(\frac{\pi}{x}\right)=0.2$ have a finite number of zeros on the interval $(-\infty, 0]$?

11. How many zeros does $\sin\left(\frac{\pi}{x-0.1}\right)=0.2$ have on the interval $(-\infty, 0]$?

12. How many zeros does $\sin\left(\frac{\pi}{x-0.1}\right)=0.2$ have on the interval $[0, \infty)$?)

13. Solve $\dfrac{\left(\sqrt[3]{\dfrac{x\sqrt{45+x\sqrt[4]{14+x\dfrac{\sqrt{|265-x|}}{6}-39}}}{8}+x}\right)^2}{32}+1496=2008$

14. Let k be the least positive integer that is divisible by each of the first ten natural numbers. Find $\left(\dfrac{k}{2002}+2204\right)^{\frac{1}{3}}$.

15. What is the exact sum of all the intercepts of the graph of $f(x)=30x^4-191x^3+258x^2+144x-160$?

16. Find the exact value of $\sum_{k=1}^{\infty}\dfrac{k}{3^k}$.

17. Find $\sum_{n=0}^{99}\cos\left[(0.05+0.1n)^2\right]$. Round to six decimal places.

18. How many times does the function $f(x)=(5-x)\cos(2\pi x)$ cross the x-axis on the interval [0, 5]?

19. $\sqrt{3\sqrt{3\sqrt{3\ldots}}}=$

20. How many pairs of integers (*a*, *b*) are there for which $a^2-b^2=625$.

21. Create a graph in polar coordinates of a flower with 8 petals where each petal is 2 units long and a tip of a petal intersects with the positive y-axis. Show the graph to an instructor and be sure to set the graphing window and axes scales so that the instructor can readily see the lengths of the petals, the number of petals, and their placement with regard to the axes (bring your answer sheet with you).

AREA OF A TRIANGLE:

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) can be determined by evaluating the following determinant:

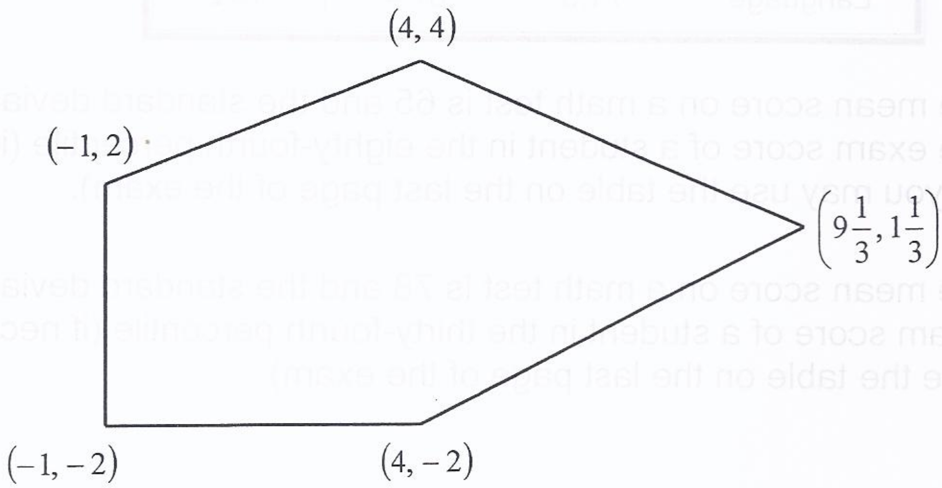
$$Area = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Where the $(\pm)$ symbol indicates that the appropriate sign should be chosen to yield a positive area. Find the areas of the following triangles, determined by the given points. Round answers to the nearest tenth if necessary.

22. Triangle's vertices: $(-2, 2)$, $(7, 5)$, and $(10, -10)$.

23. Triangle's vertices: $(-\sqrt{17}, \sqrt{23})$, $(e^3, -4)$, and $\left(-\frac{1}{2}, -10\right)$.

24. Find the area of the region defined be the vertices given below (note: the shape is not drawn to scale).



25. Find all values of *y* for which the triangle with the following vertices has an area of 100: $(-1, 5)$, $(3, -2)$, $(4, y)$.

STATISTICS:

26. Given the following test scores, find the mean, median, mode, and standard deviation of the data. Then find the z-score for the student who scored 72 on her exam.

    76, 92, 63, 85, 89, 90, 75, 67, 82, 97, 89, 72, 88, 90

27. The average score on a math test for 16 students was 66. Six more students took the test. The average of these six students was 74. What was the average for all the students (Round to the tenths place)?

28. On which test did Mrs. Jones' students perform the best compared to the national averages?

Test	Class Average	National Average	Standard Deviation
Reading	76.4	72.1	14.1
Mathematics	81.5	78.2	11.3
Science	78.9	77.2	13.7
Language	71.3	67.4	15.2

29. Assume the mean score on a math test is 65 and the standard deviation is 10. Find the exam score of a student in the eighty-fourth percentile (if necessary, you may use the table on the last page of the exam).

30. Assume the mean score on a math test is 78 and the standard deviation is 6. Find the exam score of a student in the thirty-fourth percentile (if necessary, you may use the table on the last page of the exam).

(#31 - 35) The table gives the amount of the federal debt (in billions of dollars) for various years.

Let $D = f(t)$ be the federal debt (in billions of dollars) at t years since 1960.

Year	Debt ($ billions)
1960	290
1965	321
1970	389
1975	577
1980	930
1985	1946
1990	3233
1995	4974
2000	5674
2005	7933

Source: Bureau of Public Debt

31. Find the "best-fit" model (regression equation) for $f(t)$ (round all constants to four decimal places).

32. Predict the federal debt in 2010 (round to the nearest billion).

33. If the federal debt were paid off in 2010 by each U.S. citizen contributing an equal amount of money, how much would each person have to pay. Assume the U.S. population will be about 309 million in 2010 and use the rounded dollar amount you found above.

34. Predict the federal debt in 2050 (and again round to the nearest billion).

35. If the federal debt were paid off in 2050 by each U.S. citizen contributing an equal amount of money, how much would each person have to pay? Assume the population will be about 394 million in 2050 and use the rounded dollar amount you found above.

YOU WILL USE $f(t)$ LATER SO YOU MAY NOT WANT TO DELETE IT FROM YOUR CALCULATOR.

(#36 - 40) The table gives the percentage of the U.S. Federal debt owed to foreigners for various years.

Let $p(t)$ be the percentage of U.S. debt owed to foreigners at t years since 1980.

Year	Percent
1980	17
1985	15
1990	18
1995	22
2000	31
2005	45

Source: White House Office of Management

36. Find a "best-fit" model (regression equation) model for $p(t)$. Round all constants to four decimal places.

37. Use $p(t)$ to predict the percentage of U.S. debt owed to foreigners in 2011. Round to the nearest hundredths place.

38. Use the model $D = f(t)$ from the precious page to predict the federal debt in 2011 (round to the nearest billion).

39. How much money will the U.S. Federal Government owe to foreigners in 2011? Round to the nearest billion.

40. In which year will all of the federal debt be owed to foreigners? Round up the nearest year.

ALGEBRA TOPICS:

Unless you are directed to do otherwise, round irrational numbers to the nearest hundredth. Do not round rational numbers.

41. Use your calculator to find the equation of the quadratic function that passes through the following points (do not round):

(–9, –49), (–5, 3), and (10, –144)

42. Use your calculator to find the equation of the cubic function that passes through the following points (do not round):

(–2, –72), (2, –20), (4, -351), and (6, –712)

43. Use your calculator to find the equation of the exponential function passing through the points (round constants to the nearest tenth):

(2, 10.368), (5, 60.466)

44. Solve the following system of equations (round to the nearest hundredth).

$$\begin{aligned} 3.2x - 5.7y &= 4 \\ -2.4x + 3.8y &= 7 \end{aligned}$$

45. Solve the following system of equations (write rational solutions in fractional notation, and round irrational solutions to the nearest tenth).

$$\begin{aligned} 3x - y &= 0 \\ 2y + z &= 5 \\ 7x - z &= 1 \end{aligned}$$

46. Solve the quadratic inequality, and write your solution (if it exists) in interval notation (round to the nearest tenth if necessary).
$10x^2 + 96x + 180 > 17.2$

47. Find all points of intersection of the graphs of the given equations (round to the nearest hundredth if necessary).

$$(x-3)^2 + (y-2)^2 = 9$$
$$(x-3)^2 - y = 4$$

48. Solve the following equation (if necessary, round to the nearest hundredth).

$$x^4 + 0.498x^3 - 4.604x^2 - 0.376x + 5.677 = 0$$

49. Factor the following polynomial completely (use only integers – no fractions and no rounding).

$$x^6 + 5x^5 - 87x^4 + 43x^3 + 902x^2 - 672x - 1440$$

CRYPTOLOGY:

A cryptogram is a message written according to a secret code. Matrix multiplication can be used to encode and decode messages. First we assign a number to each letter of the alphabet (A = 1, B = 2, C = 3, …, Z = 26) and assign the number 0 to a blank space. Then any message is converted to numbers and partitioned into uncoded row matrices, each with the appropriate number of entries.

For example, if our encoding matrix is a 3X3 matrix, then we would partition the message "MEET ME MONDAY" into the following 1X3 row matrices.

[13 5 5] [20 0 13] [5 0 13] [15 14 4] [1 25 0]

To encode the message we might use the matrix $A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & -4 \\ -2 & -3 & 5 \end{bmatrix}$ as follows

Uncoded Message	Encoding Matrix A	Coded Message

$$\begin{bmatrix} 13 & 5 & 5 \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & -4 \\ -2 & -3 & 5 \end{bmatrix} = \begin{bmatrix} -8 & -10 & 31 \end{bmatrix}$$

50. Given the encoding matrix $A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & -4 \\ -2 & -3 & 5 \end{bmatrix}$, encode the following message.

CALCULUS IS FUN

51. Given that the encoding matrix $A=\begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & -4 \\ -2 & -3 & 5 \end{bmatrix}$ was used to create the following coded message, decode the message.

16 15 12 9 20 9 3 19 0 3 1 14 0 2 5 0 4 9 22 9 19 9 22 5

52. Draw a motorcycle moving across your graphing window. Show it to one of the instructors (bring your answer sheet with you).

Z score	Percentile
3.7	99.99
3.65	99.99
3.6	99.98
3.55	99.98
3.5	99.98
3.45	99.97
3.4	99.97
3.35	99.96
3.3	99.96
3.25	99.95
3.2	99.93
3.15	99.91
3.1	99.91
3.05	99.89
3	99.87
2.95	99.83
2.9	99.83
2.85	99.79
2.8	99.74
2.75	99.69
2.7	99.69
2.65	99.62
2.6	99.53
2.55	99
2.5	99
2.45	99
2.4	99
2.35	99
2.3	99
2.25	99
2.2	99
2.15	98
2.1	98
2.05	98
2	98
1.95	97
1.9	97

Z score	Percentile
1.85	97
1.8	96
1.75	96
1.7	96
1.65	95
1.6	95
1.55	94
1.5	94
1.45	93
1.4	92
1.35	91
1.3	91
1.25	90
1.2	88
1.15	87
1.1	87
1.05	86
1	84
0.95	82
0.9	82
0.85	81
0.8	79
0.75	77
0.7	77
0.65	75
0.6	73
0.55	70
0.5	70
0.45	68
0.4	66
0.35	63
0.3	61
0.25	61
0.2	58
0.15	55
0.1	55
0.05	53

Z score	Percentile
0	50
-0.05	47
-0.1	45
-0.15	45
-0.2	42
-0.25	39
-0.3	39
-0.35	37
-0.4	34
-0.45	32
-0.5	32
-0.55	30
-0.6	27
-0.65	25
-0.7	25
-0.75	23
-0.8	21
-0.85	19
-0.9	19
-0.95	18
-1	16
-1.05	14
-1.1	14
-1.15	13
-1.2	12
-1.25	10
-1.3	10
-1.35	9
-1.4	8
-1.45	7
-1.5	7
-1.55	6
-1.6	5
-1.65	5
-1.7	5
-1.75	4
-1.8	4

Z score	Percentile
-1.85	3
-1.9	3
-1.95	3
-2	2
-2.05	2
-2.1	2
-2.15	2
-2.2	1
-2.25	1
-2.3	1
-2.35	1
-2.4	1
-2.45	1
-2.5	1
-2.55	1
-2.6	0.47
-2.65	0.38
-2.7	0.38
-2.75	0.31
-2.8	0.26
-2.85	0.21
-2.9	0.21
-2.95	0.17
-3	0.13
-3.05	0.11
-3.1	0.11
-3.15	0.09
-3.2	0.07
-3.25	0.05
-3.3	0.05
-3.35	0.04
-3.4	0.03
-3.45	0.03
-3.5	0.03
-3.55	0.02
-3.6	0.02
-3.65	0.01